Solutions for 2015 Algebra II with Trigonometry Exam

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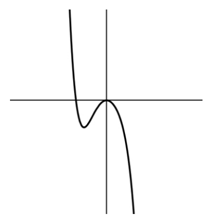
1. For $f(x) = 6x - 4(2x - 1)^2 + 5$, find $f(2)$.

   Solution: The notation $f(2)$ tells us to evaluate the function $f(x)$ at $x = 2$. Thus we have

$$\begin{aligned} f(2) &= 6(2) - 4(3)^2 + 5 \\ &= 12 - 4 \cdot 9 + 5 \\ &= -24 + 5 \\ &= -19 \end{aligned}$$

   Therefore the answer is $\boxed{-19}$.

2. Which of the following could be the leading term of the polynomial whose graph is as pictured?



   Solution: The end behavior of the graph indicates it has an negative, odd degree leading term. Though the first thing that likely comes to mind is cubic, any odd power could yield this graph. Thus the best answer from the available choices is $\boxed{-2x^5}$.

3. What is the largest solution of the equation $2x^3 - 5x^2 = 8x - 20$?

   Solution: To solve this equation, we can move everything to the left side, factor by grouping, and then use the zero-product property to solve.

$$\begin{aligned} 2x^3 - 5x^2 - 8x + 20 &= 0 \\ x^2(2x - 5) - 4(2x - 5) &= 0 \\ (x^2 - 4)(2x - 5) &= 0 \\ (x - 2)(x + 2)(2x - 5) &= 0 \end{aligned}$$

   Thus the solutions are $x = 2, -2, 5/2$ and so the largest solution is $5/2$. So the correct answer is $\boxed{\text{None of These}}$.

   Note: Because of the presence of the None of These answer choice, it is not enough to simply check the available answer choices and choose the largest one that works.

4. Which of the following functions is/are equal to $f(x) = 4^x$?

   Solution: It helps to start with 4^x and think about exponent laws. We can write 4 as 2^2 and so $4^x = (2^2)^x$. When we have a power raised to another power, we multiply the powers. Thus $4^x = (2^2)^x = 2^{2x}$. So we know that whatever the answer is, it must include III. Again by exponent laws, $(2^x)^2 = 2^{2x}$, and so we also know that II and III are equivalent. To see that I is not equivalent to these, you'll need to remember that when you have nested exponents like this (one value raised to another, raised to another, etc.), the order of operations works from the top down. That is, for example, if you had 2^{2^3}, you would first perform the 2^3 to get 8 and then perform 2^8. Thus by simple example (using $x = 3$) you can see that I would yield 2^8, while II and III both give 2^6. Thus the answer is $\boxed{\text{II and III only}}$.

5. The number of values satisfying the equation $\dfrac{2x^2 - 10x}{x^2 - 5x} = x - 3$ is:

   Solution: To solve this rational equation, multiply both sides by $x^2 - 5x$ and then solve the resulting cubic equation.

$$\begin{aligned}
2x^2 - 10x &= (x - 3)(x^2 - 5x) \\
2x^2 - 10x &= x^3 - 8x^2 + 15x \\
x^3 - 10x^2 + 25x &= 0 \\
x(x^2 - 10x + 25) &= 0 \\
x(x - 5)^2 &= 0
\end{aligned}$$

   Thus we get $x = 0$ and $x = 5$ as two possible solutions. However, our original equation had domain restrictions. Checking both answers shows that both make the denominator of the rational expression on the left hand side of the equation 0, and thus are not solutions to the equation. Therefore the equation as $\boxed{0}$ solutions.

6. The difference quotient of a function $f(x)$ is the quotient $\dfrac{f(x+h) - f(x)}{h}; h \neq 0$. Find the difference quotient of $f(x) = \frac{3}{x}$.

   Solution: Recall that $f(x+h)$ means that we evaluate the function f at $x + h$. Thus plugging in $x + h$ where we see x, we get that $f(x+h) = \frac{3}{x+h}$. Now we can put this together in the difference quotient formula.

$$\frac{\frac{3}{x+h} - \frac{3}{x}}{h} = \frac{\frac{3x}{x(x+h)} - \frac{3(x+h)}{x(x+h)}}{h} = \frac{\frac{3x-(3x+3h)}{x(x+h)}}{h} = \frac{\frac{3x-3x-3h}{x(x+h)}}{h}$$

   Combining denominators, and simplifying further, we get

$$\frac{\frac{3x-3x-3h}{x(x+h)}}{h} = \frac{-3h}{x(x+h)h} = \frac{-3}{x(x+h)}$$

   Therefore the answer is $\boxed{\frac{-3}{x(x+h)}}$.

7. In a warehouse, a stack of 6 foam mattress toppers are piled up. Each mattress topper is originally 3

$$
\begin{aligned}
(3x - 10)(x + 1) &= 10 \\
3x^2 - 7x - 10 &= 10 \\
3x^2 - 7x - 20 &= 0 \\
(3x + 5)(x - 4) &= 0
\end{aligned}
$$

Therefore there are two solutions $x = -\frac{5}{3}$ and $x = 4$. Thus the solution is $-\frac{5}{3} + 4 = \boxed{\frac{7}{3}}$

9. The student council is made up of four sophomores, two juniors and three seniors. A yearbook photographer would like to line up all members of the student council in a line for a picture. How many different pictures are possible if students in the same grade stand beside each other?

   Solution: Since the students of the same grade need to stand together, we need to arrange four things:

and so $x^3 + \frac{1}{x^3} = \boxed{18}$.

13. Find the slope of the line perpendicular to the line connecting the two points $(\frac{1}{4}, -\frac{1}{3})$ and $(-\frac{1}{6}, 1)$.

Solution: The formula for the slope of a line between two points is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Thus, the slope is:

$$m = \frac{1 - \frac{-1}{3}}{-\frac{1}{6} - \frac{1}{4}} = \frac{\frac{4}{3}}{-\frac{5}{12}} = \frac{4}{3} \cdot -\frac{12}{5} = -\frac{16}{5}$$

Now, the question asks for the slope of a line perpendicular to the line connecting the points. The slope of a perpendicular line is the negative reciprocal of the slope of the original line, and so the slope of the perpendicular line is $\boxed{\frac{5}{16}}$.

14. In the senior class at a particular high school, 45 students are taking calculus, 52 students are taking physics, and 21 students are taking both calculus and physics. If there are 200 people in the senior class, what is the probability that a randomly selected student is taking calculus or physics?

Solution: To solve this problem, we can either fill out a Venn Diagram, or use the Union Rule. The union rule states that $P(E \cup F) = P(E) + P(F) - P(E \cap F)$, for events E and F. Since we are looking for the probability of taking calculus or physics, we are solving for $P(E \cup F)$ in the equation. Now, $P(\text{taking calculus}) = 45/200$, $P(\text{taking physics}) = 52/200$ and $P(\text{taking calculus and physics}) = 21/200$. Thus

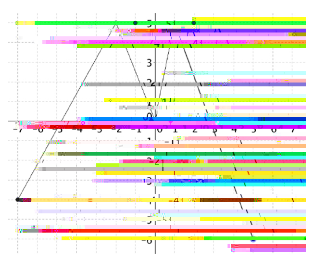
$$\begin{aligned} P(\text{taking calculus or physics}) &= P(\text{taking calculus}) + P(\text{taking physics}) - P(\text{taking calc and physics}) \\ &= 45/200 + 52/200 - 21/200 \\ &= 76/200 \end{aligned}$$

Thus $P(\text{taking calculus or physics}) = 76/200$ or $\boxed{0.38}$.

15. To which of the following expressions is $\sqrt[8]{x^8} + \sqrt[7]{x^7}$ equal to for all real negative values of x?

Solution: Recall that $\sqrt[n]{x^n} = |x|$ for even n and $\sqrt[n]{x^n} = x$ for odd n. Therefore for all values of x, $\sqrt[8]{x^8} = |x|$ and $\sqrt[7]{x^7} = x$. But since $x < 0$ in this question, we know that $|x| = -x$. Thus $\sqrt[8]{x^8} + \sqrt[7]{x^7} = |x| + x = -x + x = 0$. Therefore the correct answer, for negative values of x, is $\boxed{0}$.

16. The graph of the function f is shown below. How many solutions does the equation $f(f(x)) = 5$ have?



Solution: This problem will involve solving two equations: for which z are $f(z) = 5$ and then for which values of x do $f(x) = z$

$(-\infty, 2)$, $[2, 3)$ and $[3, \infty)$. We will solve the equation in each of these three intervals, and that will give us all solutions. For $(-\infty, 2)$, we know that both $|x - 2| = 2 - x$ and $|x - 3| = 3 - x$. Thus the equation simplifies to $2 - x + 3 - x = 1$. Further work with this gives $5 - 2x = 1$, which gives a single answer of $x = 2$. [Technically this is in the next interval, but we can count it either place. The choice of where to include 2 was arbitrary]

Now for the interval $[2, 3)$, we know that $|x - 2| = x - 2$ and $|x - 3| = 3$

So the equation of the rst line is $y = 3x$ 27. Similar3

Now, you can see that each of the three terms share an $x^{3=6}$. We will factor it out:

$$x^{3=6}(2x^{2=6} \quad 5x^{1=6} + 2) = 0$$

The factor $x^{3=6}$ will result in a solution of 0, but since the problem asked for nonzero solutions, we can ignore this factor. This leaves us with a quadratic-like equation of

$$2x^{2=6} \quad 5x^{1=6} + 2 = 0$$

Let $z = x^{1=6}$. Then we can write our equation as

$$2z^2 \quad 5z + 2 = 0$$

This factors as $(2z \quad 1)(z \quad 2) = 0$, which gives solutions of $z = {}^{1}$

30. Find the inverse function $f^{-1}(x)$ of the function $f(x) = \dfrac{x+2}{x+3}$.

Solution: One method of nding an inverse function is to swap x and y in the original function, and then re-solve for y. Thus we have

$$
\begin{aligned}
x &= \tfrac{y+2}{y+3} && \text{Swap x and y} \\
x(y+3) &= y+2 && \text{Multiply both sides by } y+3 \\
xy+3x &= y+2 && \text{Distribute} \\
xy \quad y &= \quad 3x+2 && \text{Rearrange} \\
y(x \quad 1) &= 2 \quad 3x && \text{Factor out a } y \\
y &= \tfrac{2-3x}{x-1}
\end{aligned}
$$

Thus the answer is $0 + 3 + \frac{4}{5} + \frac{9}{5} = \boxed{\frac{28}{5}}$

33. Let $f(x)$ be a function such that $f(x$

36. Define an operation # on pairs of real numbers as

$$(x_1, y_1) \# (x_2, y_2) = (x_1^2 x_2^2, y_1 y_2).$$

Which of the following could **not** equal $(x_1, y_1) \# (x_2, y_2)$ for any real numbers x_1, y_1, x_2, y_2?

Solution: Notice first that the first term in $(x_1^2 x_2^2, y_1 y_2)$ is a perfect square: $(x_1 x_2)^2$. Since this operation is defined only on real numbers, this must be positive. Thus the pair $\boxed{(-1, 0)}$ cannot be $(x_1, y_1) \# (x_2, y_2)$ for any real numbers numbers x_1, y_1, x_2, y_2.

37. The straight lines $-ax + \frac{1}{2}y = 1$ and $(a + 1)x + y = 1$ are parallel to each other. Find the value of the constant a.

Solution: Since we are told the lines are parallel, we know they have the same slope. First, we need to find the slopes. Then we can set them equal to each other and solve the resulting equation. Solving for y in both, we see that the first line can be written as $y = 2ax + 2$ and the second line as $y = -(a+1)x + 1$. Thus the slopes are $2a$ and $-(a + 1)$ respectively. Setting these equal to each other we get the equation $2a = -(a + 1) = -a - 1$ which gives $a = \boxed{-\frac{1}{3}}$.

38. A box contains 1 white, 3 purple, and 2 gold balls. A second box contains 2 purple and 2 gold balls. One ball is selected at random from each box. What is the probability they are the same color?

42. Which of the following functions has a graph with exactly one hole, occurring when $x = 1$ and exactly one vertical asymptote, occurring when $x =$ 7?

Solution: Recall that a hole occurs at $x = a$ if a is a zero of the denominator and the numerator. An asymptote occurs at $x = b$

47. Divide $1 \quad 7i$ by $6 \quad 2i$. Put your answer in $a + bi$ form.

Solution: To get the answer into $a + bi$ form, we must multiple both the numerator and denominator by the complex conjugate, $6 + 2i$, of the denominator.

$$\frac{1 \quad 7i}{6 \quad 2i} \frac{6 + 2i}{6 + 2i} = \frac{6 \quad 40i \quad 14i^2}{36 \quad 4i^2} = \frac{6 \quad 40i + 14}{36 + 4} = \frac{20 \quad 40i}{40} = \frac{1}{2} \quad i$$

Thus the answer is $\boxed{\frac{1}{2} \quad i}$.

48. Find the sum of all solutions to the equation

$$(x^3 + x^2 + 5x \quad 11)^2 \quad (4x^2 \quad 4x + 16)^2 = 0.$$

Solution: Notice that this equation is the di erence of two squares. Recall that for any a, b, $a^2 \quad b^2 = (a + b)(a \quad b)$. We can factor and simplify the equation:

$$\begin{aligned} (x^3 + x^2 + 5x \quad 11) + (4x^2 \quad 4x + 16) \quad (x^3 + x^2 + 5x \quad 11) \quad (4x^2 \quad 4x + 16) &= 0 \\ x^3 + 5x^2 + x + 5 \quad x^3 \quad 3x^2 + 9x \quad 27 &= 0 \\ x^2(x + 5) + 1(x + 5) \quad x^2(x \quad 3) + 9(x \quad 3) &= 0 \\ (x^2 + 1)(x + 5)(x^2 + 9)(x \quad 3) &= 0 \end{aligned}$$

The solutions to the four factors are, respectively, $i, \quad i$, $\quad 5$, $3i, \quad 3i$, 3. Finally, the sum of all solutions is $\boxed{\quad 2}$.

49. Two committees consisting of 3 and 5 people, respectively, are to 0t9615 Td [(7.)]TJ0 g 0 G -47.234t,p pand 5 respec04